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Dr. T. THIMMAIAH INSTITUTE OF TECHNOLOGY

(Affiliated to Visvesvaraya Technological University, Belgaum)

OORGAUM, K.G.F. - 563 120.

Dept. of E.C.E. Sl.No. .... Dr. T.T.I.T., K.G.F.

Assignment / Quiz / Class Test

Certificate

This is to certify that Mr. / Ms. Kanisha.R bearing USN. No. 1GV19EC017 has satisfactorily completed the course of Tests and assignments as prescribed by Visvesvaraya Technological University for III Semester B.E./ ~~M.Tech~~, Degree in ECE Branch / Specialization for the academic year 2020-2021 for the Subject Digital System Design and Code 18EC34

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23/5/21	A/Q/CT	10	9.3	(signature)	Kanisha.R
16/7/21	A/Q/CT	10	9	(signature)	Kanisha.R
9/8/21	A/Q/CT	10	8.6	(signature)	Kanisha.R

19/1/22
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Head of the Department
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1. Explain Serial Adder with accumulator and design a Control circuit for Serial adder.

→ In this Section we will design a control circuit for a Serial adder with an accumulator. fig 18.1 shows a block diagram for the adder. Two shift registers are used to hold the 4-bit numbers to be added. X and Y. The X register serves as an

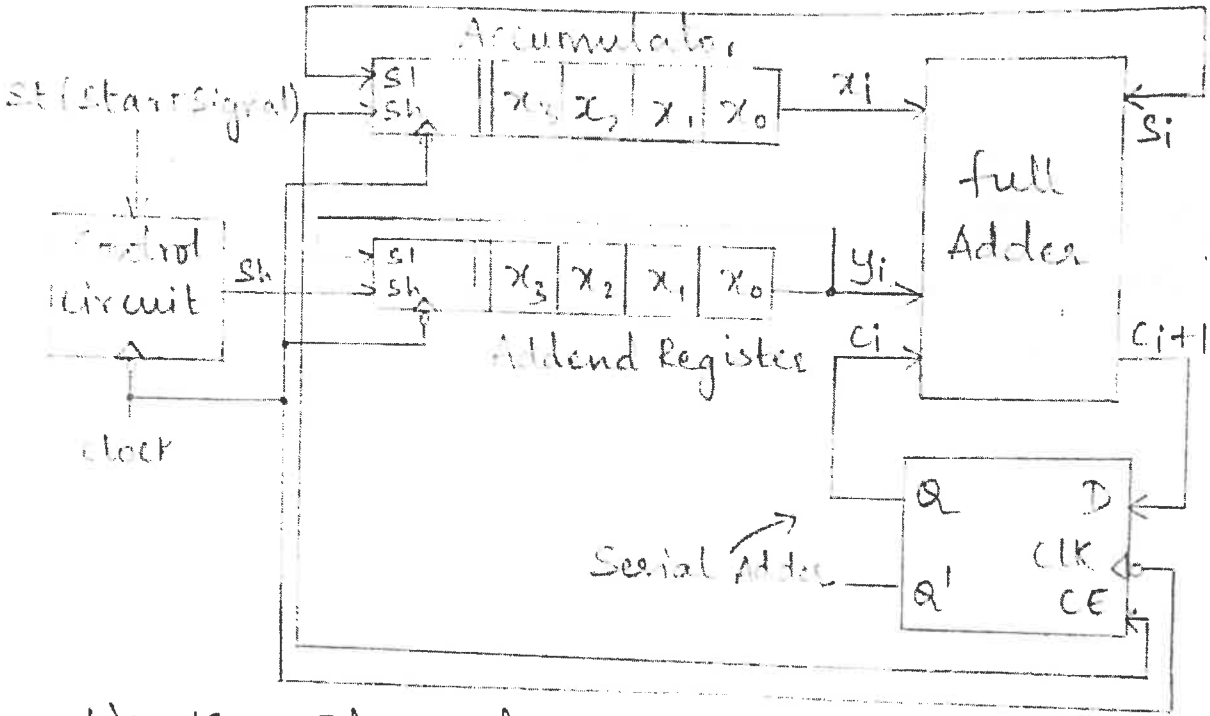


fig 18.1 Block diagram for Serial Adder with accumulator.

accumulator and the Y register serves as an addend register.

* when the addition is completed, the contents of the X register are replaced with the sum of X and Y.

* The addend register is Connected as a cyclic shift register so that after shifting four times it is back in its original state, and the number Y is not lost.

: The box at the left end of each shift register shows the inputs: sh (Shift Signal), SI (Serial input) and clock, when Sh=1 and an active clock edge occurs, SI is entered into x_3 (or y_3) at the same time as the contents of the register are shifted one place to the right.

* The additional connections required for initially loading the X and Y registers and clearing the carry flip-flop are not shown in the block diagram

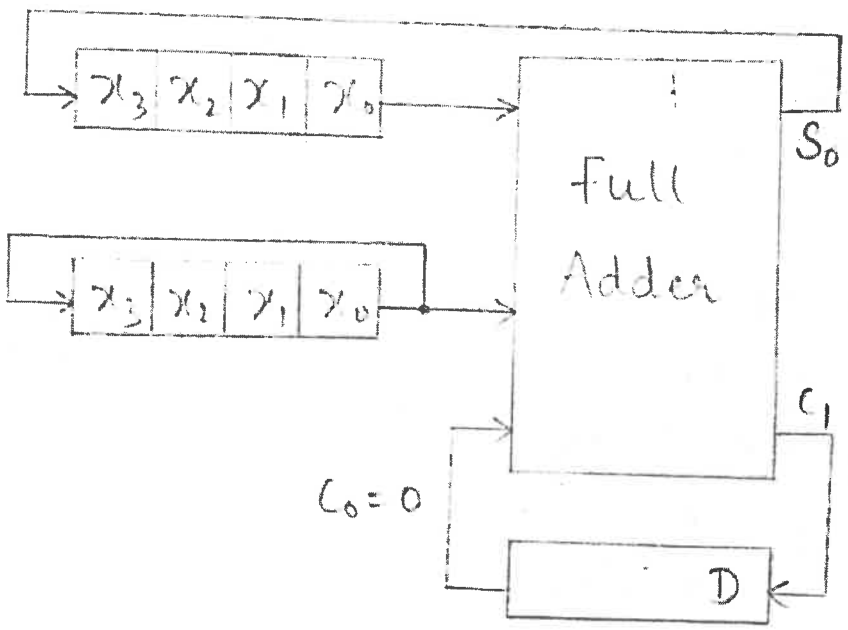
* The serial adder, highlighted in blue in the diagram is the same as the one in fig 13.12, expect the D flip-flop has been replaced with a D flip-flop with clock enable. At each clock time, one pair of bits is added. Because the full adder is a combinational circuit, the sum and carry appear at the full adder output after the propagation delay.

* when Sh=1, the falling clock edge shifts the sum bit into the accumulator, stores the carry bit in the carry flip-flop, and rotates the addend register one place to the right.

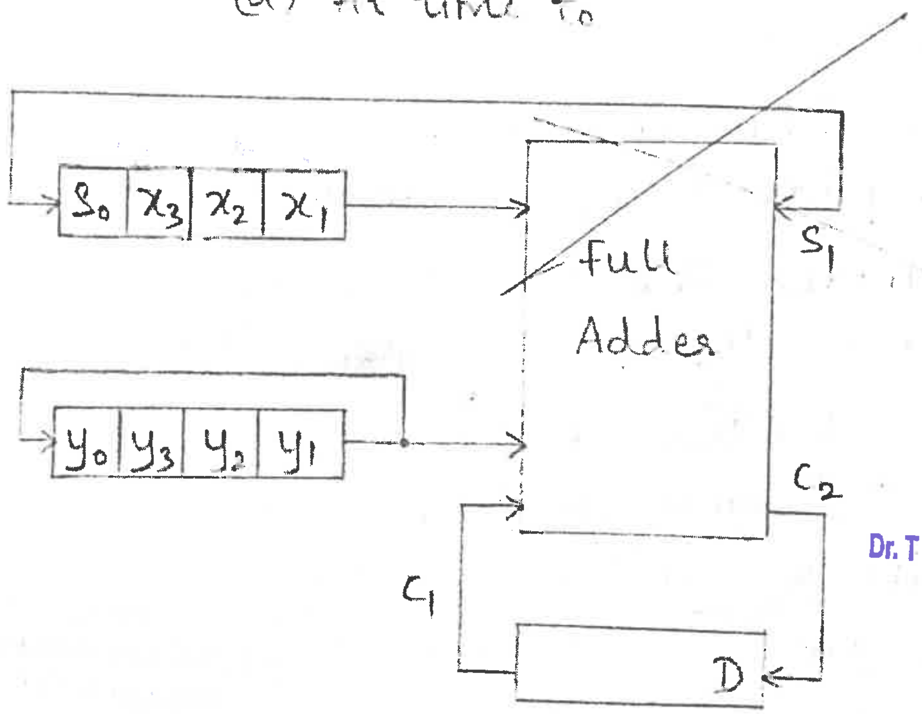
* Because Sh is connected to CE on the flip-flop, the carry is only updated when shifting occurs.

* fig 18.2 illustrates the operation of the adder. Shifting occurs on the falling clock edge when sh=1. In this fig. t_0 is the time before the first shift. t_1 is the time after the first shift, t_2 is the time after the second shift. etc. initially at time t_0, the accumulator contains X and the addend register contains Y. Because the full adder is a combinational ckt. X_0, Y_0 and C_0 are added independently of the

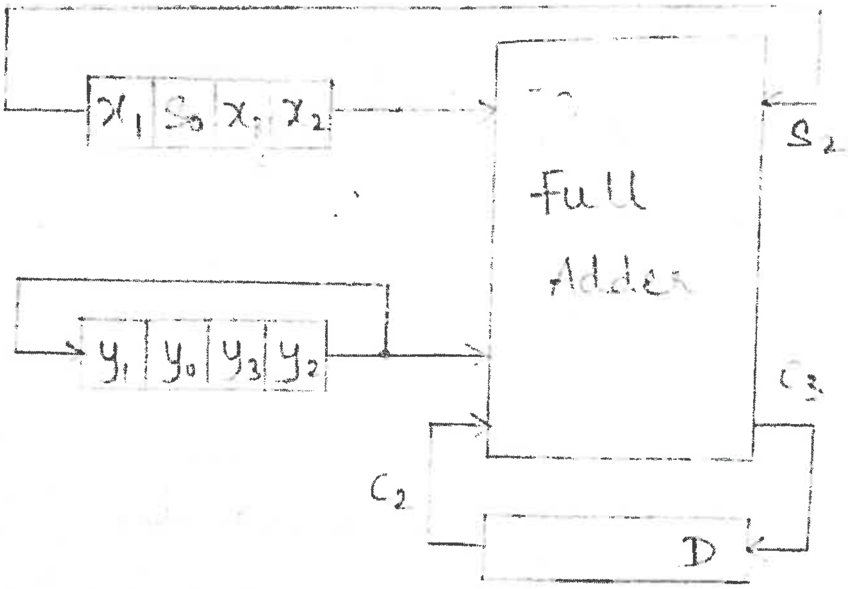
clock to form the Sum S_0 and Carry C_1. When the first falling clock edge occurs, S_0 is shifted into the accumulator and the remaining accumulator digits are shifted one position to the right. The same clock edge stores C_1 in the carry flip-flop and rotates the addend register right. The next pair of bits, x_1 and y_1, are now at the full adder input, and the adder generates the sum and carry S_1.



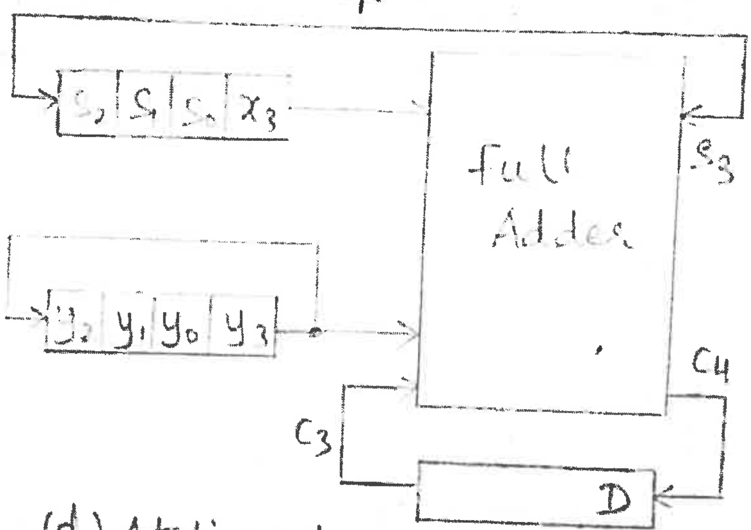
(a) At time t_0



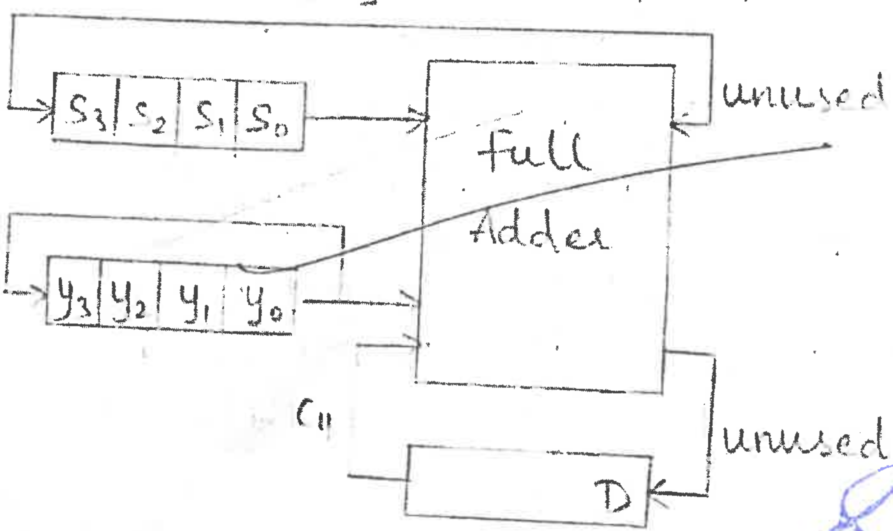
(b) At time t_1.



(c) At time t_2.



(d) At time t_3



(e) At time t_4

and C_2 as seen in fig 18-2(b). The second falling edge shifts S_1 into the accumulator, stores C_2 in the carry flip-flop and cycles the addend register right. Bits x_2 and y_2 are now at the adder input, as seen in fig 18-2(c), & the process continues until all bit pairs have been added as shown in fig 18-2(e)

Table 18-1 shows a numerical example of the serial adder operation. Initially, the accumulator contains 0101 and the addend register contains 0111. At t_0, the full adder computes $1+1+0=10$, so, $S_i=0$ & $C_i^+=1$. After the first falling clock

Table 18-1

operation of Serial Adder

	X	Y	C_i	S_i	C_i^+
t_0	0101	0111	0	0	1
t_1	0010	1011	1	0	1
t_2	0001	1101	1	1	1
t_3	1000	1110	1	1	0
t_4	1100	0111	0	(1)	(0)

edge (time t_1) the first sum bit has been entered into the accumulator, and the carry has been stored in the carry flip-flop and the addend has been cycled right. After four falling clock edges (time t_4). the sum of X and Y is the accumulator, and the addend register is back to its original state

The control circuit for the adder must now be designed so that after receiving a start signal, the control circuit will put out four shift signals and then stop. figure 18-3 shows the state graph and table for the control circuit. The circuit remains in S_0 until a start signal is received, at which time the circuit outputs $Sh=1$ & goes to S_1. Then at successive clock times, three more shift signals are put out. It will be assumed that the start signal is terminated before the circuit returns to state S_0. So that no further output occurs until another start signal is received. Dashes appear on the graph because once S_1 is reached, the circuit operation continues regardless of the value of St. Starting with the state table of fig 18-3 and using a straight binary state assignment, the control circuit equations are derived in fig 18-4

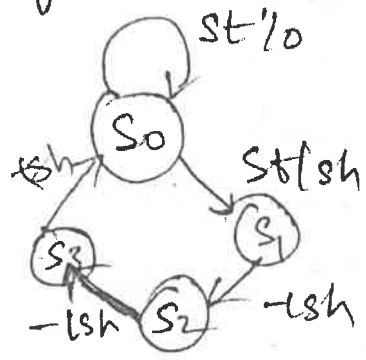
A serial processing unit, such as a serial adder with an accumulator, processes data one bit at a time. A typical serial processing unit (fig. 18-5) has two shift registers. The output bits from the shift register are inputs to a combinational circuit. The combinational circuit generates at least one output bit when the active clock edge occurs, this bit is stored in the first bit of the shift register at the same time the register bits are shifted to the right.

* The control for the serial processing unit generates a series of shift signals. When the start signal (St) is 1,

The first shift signal (Sh) is generated if the shift registers

fig 18-3

State graph for Serial Adder control.



	Next State St=0	1	Sh 0	1
S_0	S_0	S_1	0	1
S_1	S_2	S_2	1	1
S_2	S_3	S_3	1	1
S_3	S_0	S_0	1	1

fig. 18-4

Derivation of Control circuit equations

	A B	A^+B^+ 0	1
S_0	0 0	00	01
S_1	0 1	10	10
S_2	1 0	11	11
S_3	1 1	00	00

AB \ St	0	1
00	0	0
01	1	1
11	0	0
10	1	1

A^+: $D_A = A'B + AB' = A \oplus B$

AB \ St	0	1
00	0	1
01	0	0
11	0	0
10	1	1

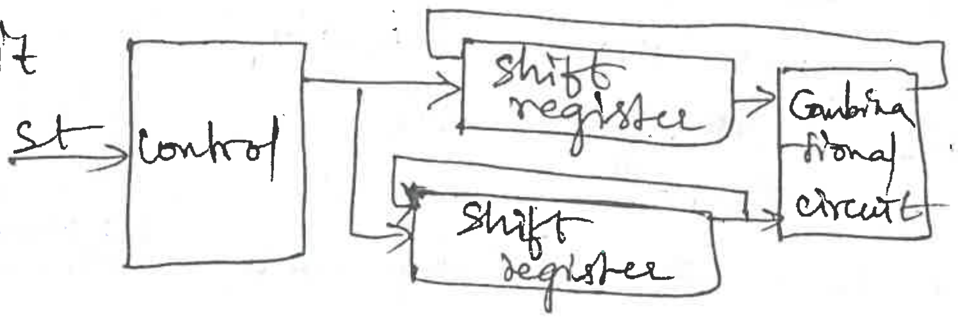
B^+: $D_B = StB' + AB'$

AB \ St	0	1
00	0	1
01	1	1
11	1	1
10	1	1

Sh: $Sh = St + A + B$

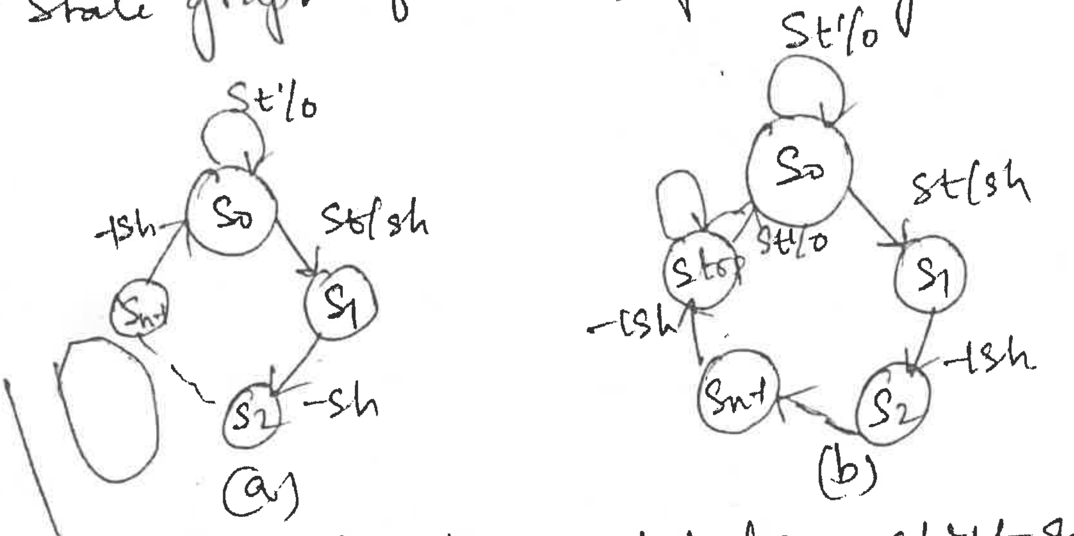
fig 18-5

Typical serial processing unit



~~fig 18-6~~

State graphs for Series processing unit



have n-~~(?)~~ bits then a total of n shift signals must be generated. If St is 1 for only one clock time, then the control state graph [fig 18-6(a)] stops when it returns to state S0. However, if St can remain 1 until after the shifting is completed, then a separate stop state is required, as shown in fig. 18-6(b). The control remains in the stop state until St returns to 0

~~3. [illegible] diagram explain parallel multiplier and design a control circuit~~

3. Explain the design of binary divider.

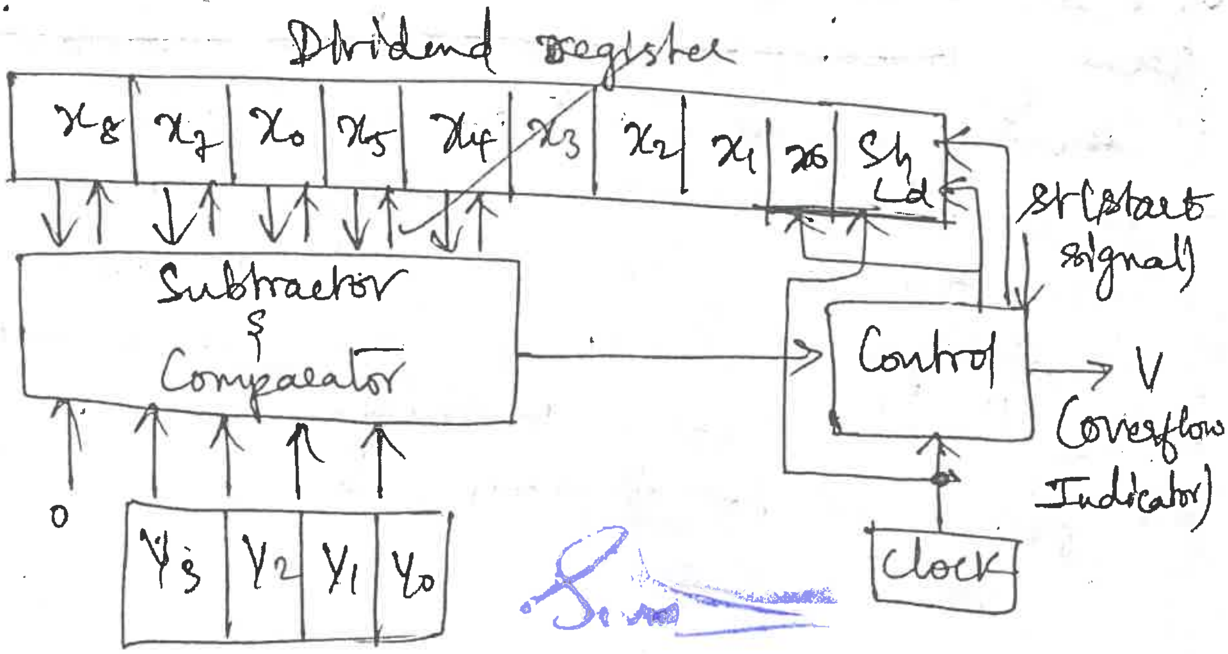
→ we will consider the design of a parallel divider for the binary numbers. As an example, we will design a circuit to divide an 8-bit dividend by a 4-bit divisor to obtain a 4-bit quotient. The foll example illustrates the division process.

```
                 1010        quotient
divisor 1101 ) 10000111      dividend
               1101
               ----
               0111
               0000
               ----
                1111
                1101
                ----
                0101
                0000
                ----
                0101         remainder
```

(135 ÷ 13 = 10 with a remainder of 5)

Just as binary multiplication can be carried out as series of add & shift operations, division can be carried out by a series of subtraction and shift operations. To construct the divider, we will use a 9-bit dividend register and a 4-bit divisor register as shown in the fig 18-10. During the division process, instead of.

fig 18-10.



Shifting the divisor to the right before each subtraction as shown in the preceding example, we will shift the dividend to the left.

* Note that an extra bit is required on the left end of the dividend register so that a bit is not lost when the dividend is shifted left.
* Instead of using a separate register end of the dividend register as the dividend is shifted left.
* The preceeding division example (135 divided by 13) is now reworked, showing the location of the bits in the registers at each clock time. Initially, the dividend and divisor are entered as follows:

0	1	0	0	0	0	1	1	1

1	1	0	1

* Subtraction cannot be carried out without a negative result, so we will shift before we subtract.
* Instead of shifting the divisor one place to the right we will shift the dividend one place to the left:

1 0 0 0 0 1 1 1 | 0 ← Dividing line b/w dividend & quotient
1 1 0 1

↑ Note that after the shift, the rightmost position in the dividend register is "empty".

* Subtraction ~~cannot~~ is now carried out & the first quotient digit of 1 is stored in the unused position of the dividend register

0 0 0 1 1 1 1 1 1 ← first quotient digit

Next, we shift the dividend one place to the left.

0 0 1 1 1 1 1 1 0
1 1 0 1

Because subtraction would yield a negative result, we shift the dividend to the left again, and the second quotient bit remains 0

0 1 1 1 1 1 1 1 | 1 0 0
1 1 0 1

Subtraction is now carried out, and the third quotient digit of 1 is stored in the unused position of the dividend register!

0 0 0 1 0 1 | 1 0 1 ← third quotient digit

A final shift is carried out and the fourth quotient bit is set to 0:

0 0 1 0 1 | 1 0 1 0
remainder | quotient

for example, if we attempt to divide 135 by 7. the initial contents of the registers would be:

0 1 0 0 0 0 1 1 1
0 1 1 1

* Because subtraction can be carried out with a nonnegative result, we should subtract the divisor from the dividend and enter a quotient bit of 1 in the rightmost place contains the least significant bit of the dividend & entering a quotient bit here would destroy that

dividend bit.

for fig. 18-10, if initially $X_8 X_7 X_6 X_5 X_4 \geq Y_3 Y_2 Y_1 Y_0$ (i.e, if the left five bits of the dividend register exceed or equal the divisor), the quotient will be greater than 15 and an overflow occurs. Note that if $X_8 X_7 X_6 X_5 X_4 \geq Y_3 Y_2 Y_1 Y_0$ is the quotient

$$\frac{X_8 X_7 X_6 X_5 X_4 X_3 X_2 X_1 X_0}{Y_3 Y_2 Y_1 Y_0} \geq \frac{X_8 X_7 X_6 X_5 X_4 0000}{Y_3 Y_2 Y_1 Y_0}$$

$$= \frac{X_8 X_7 X_6 X_5 X_4 \times 16}{Y_3 Y_2 Y_1 Y_0} \geq 16$$

* We will now design the control circuit using a one-hot assignment (see section 15.9) to implement the state graph. One flip-flop is used for each state with $Q_0 = 1$ in S_0, $Q_1 = 1$ in S_1, $Q_2 = 1$ in S_2, etc. By inspection, the next state and output equations are

$$Q_0^+ = St' Q_0 + CQ_1 + Q_5 \qquad Q_1^+ = St Q_0$$

$$Q_2^+ = C' Q_1 + C' Q_2 \qquad Q_3^+ = C' Q_2 + CQ_3$$

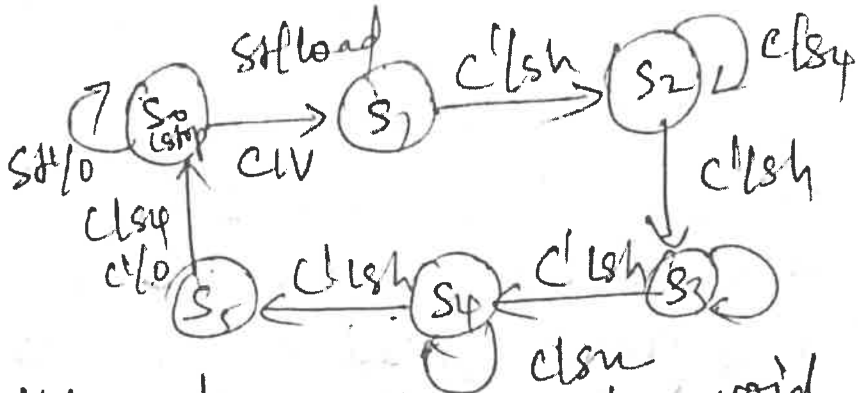
$$Q_4^+ = C' Q_3 + CQ_4 \qquad Q_5^+ = C' Q_4$$

$$load = St Q_0 \qquad V = CQ_1$$

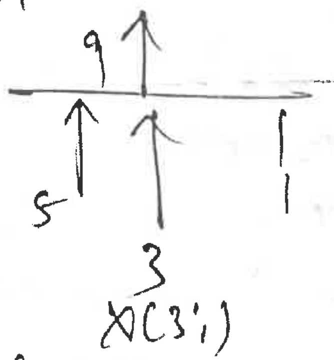
$$Sh = C'(Q_1 + Q_2 + Q_3 + Q_4) = C'(Q_0 + Q_5)'$$

$$Su = C(Q_2 + Q_3 + Q_4 + Q_5) = C(Q_0 + Q_1)'$$

fig:- State graph for divider Control ckt



* This diagram uses bus notation to avoid drawing multiple wires, when several busses are merged together to form a single bus. a bus merger is used for example, the symbol



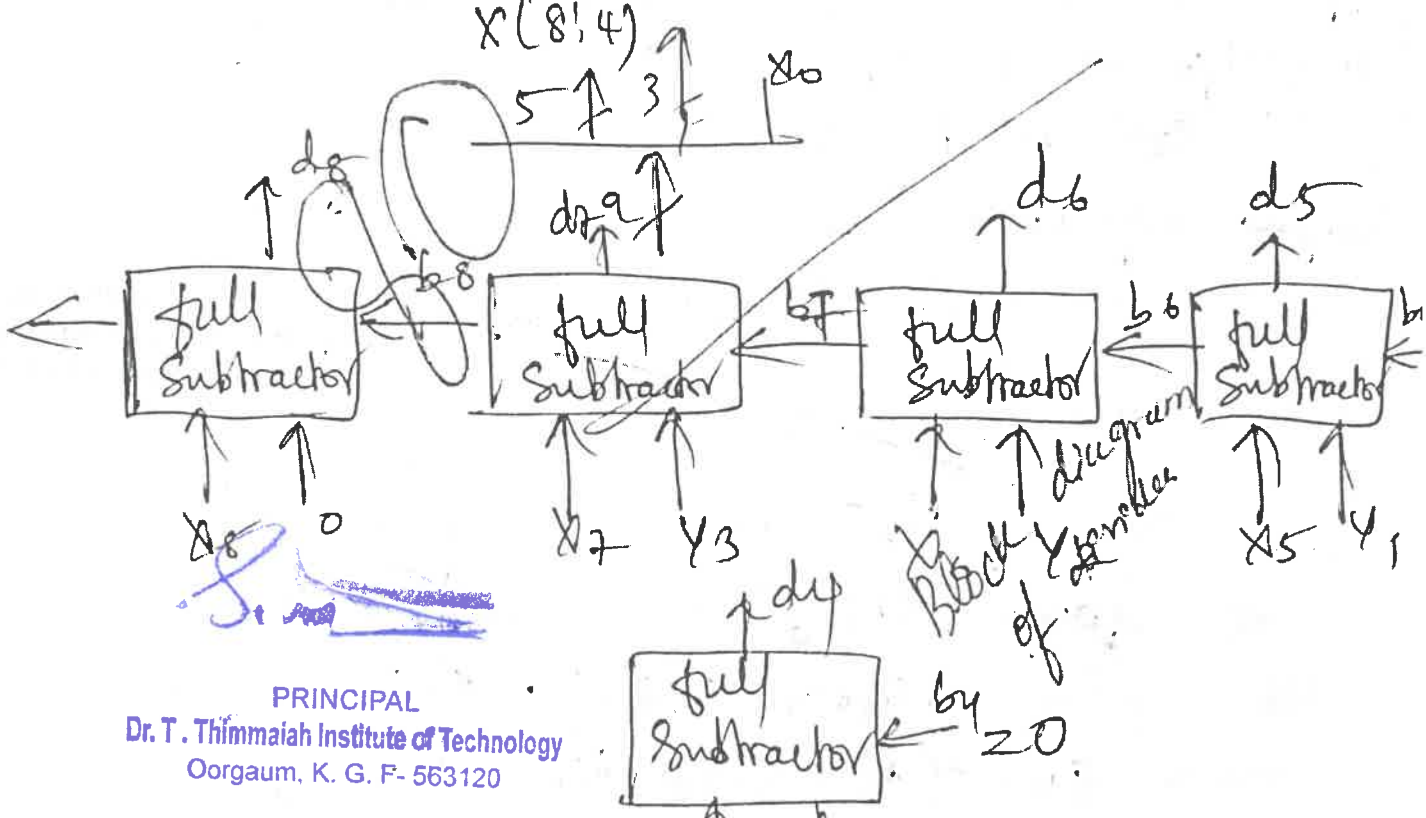


fig:- 18-12 logic diagram for 5-bit subtractor

2 With a neat block diagram explain a parallel binary multiplier and design a control circuit for binary multiplier.

* We will design a parallel multiplier for two binary numbers. As illustrated in the example in Section 1.3

* binary multiplication requires only shifting and adding

* The foll example shows how each partial product is added in as soon as it is ~~formed~~ formed. This eliminate the need for adding more than two binary numbers at a time.

```
Multiplicand ──→      1101   (13)
Multiplier   ──→      1011   (11)
                      ----
                      1101
partial   }          1101
product   }         ------
                    100111
                    0000
                   -------
                   100111
product ──→        1101
                  --------
                  10001111 (143)
```

* The multiplication of two 4-bit numbers requires a 4-bit numbers requires a 4-bit numbers requires a 4-bit ~~multiplicand~~ ~~field~~ multiplicand register a 4-bit multiplier register and an 8-bit register for the product. the product register serves as an accumulator to accumulate the sum of partial products

* Instead of shifting the multiplicand left each time before it is added as was done in the previous
* The adder calculate the sum of its inputs and when an add signal occurs
* The load signal loads the multiplier into the lower four bits of Acc and at the same time clears the upper 5 bits
* The control circuit puts out the proper sequence of add and shift signals after a start signal (St = 1) has been received.
* The multiplication example at the beginning of this section (13 X 11) is reworked below showing the location of the location of the bits in the registers at each clock time

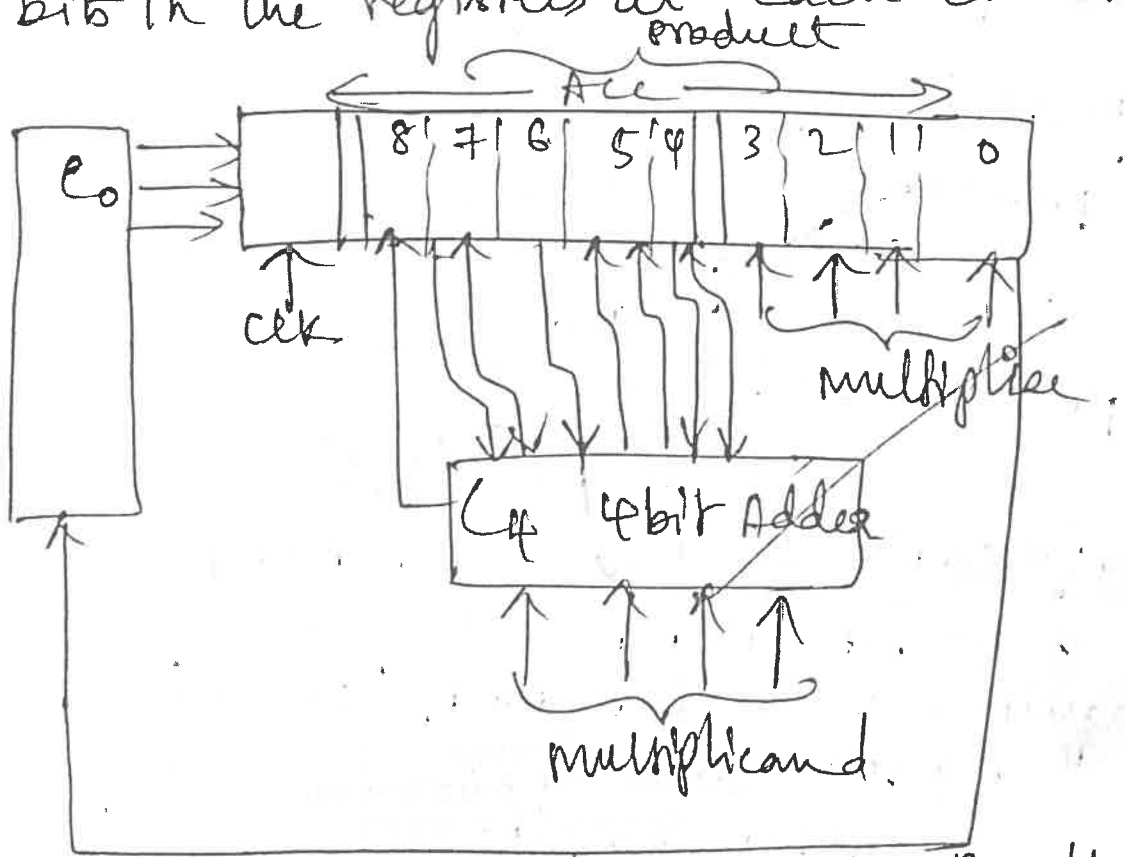
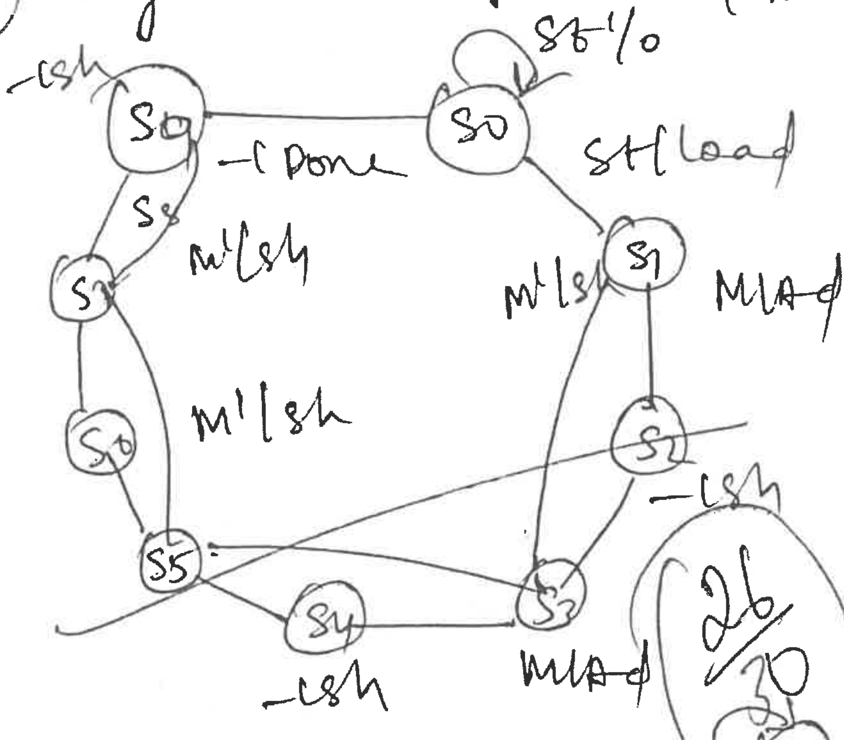


Fig: 18.7 : Block diagram for Parallel binary Multiplier

Initial contents of product register	0 0000 | 1011 ← mult
	1101 (13)
(add multiplicand because M=1)	
after addition	01101 | 1011 ← M
after shift	00110 | 1101
	1101
(skip addition because M=0)	10011 | 1101 ← M
after shift	01001 1 | 110
(add multiplicand and because M=1)	00100111 | 1 ← M
	1101
after addition	10001 1111 (143)
after shift (final answer)	010001111

dividing line b/w product & multiplier ↗



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No of pages used 16.

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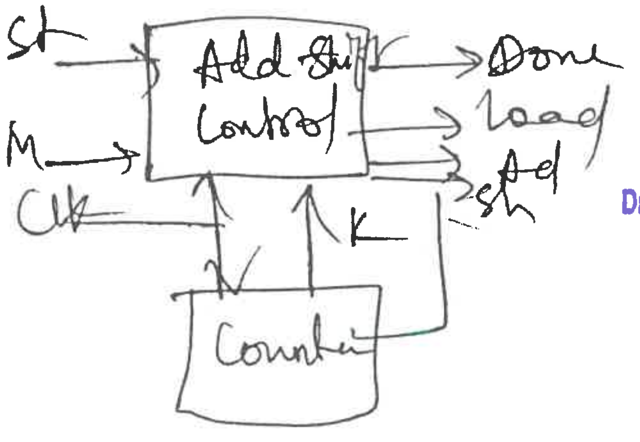

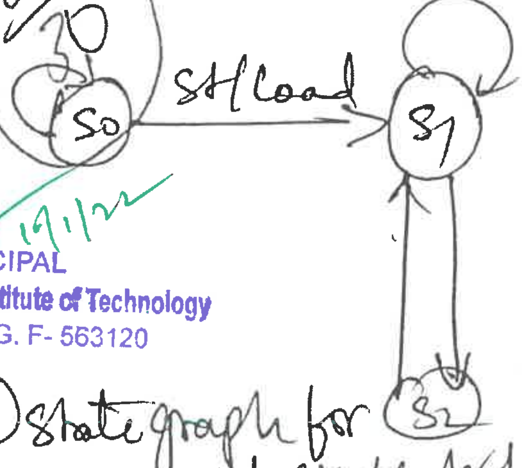

(b) state graph for add shift control

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Assignment / Quiz / Class Test

Certificate

This is to certify that Mr. / Ms. N. Vidhya Sagar *bearing* USN. *No.* [illegible] *has satisfactorily completed the course of Tests and assignments as prescribed by Visvesvaraya Technological University for* 4th *Semester B.E. / M.Tech. Degree in* Mechanical *Branch / Specialization for the academic year* 2020-21 *for the Subject* Complex analysis, Probability & Statistical Methods *and Code* 18MAT41

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11/5/21	A/Q/CT	10	10	[signature]	[signature]
14/7/21	A/Q/CT	10	8	[signature]	[signature]
05/8/21	A/Q/CT	10	10	[signature]	[signature]

[signature] 14/1/22
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Signature of HOD

2nd Internal Class Test

(2) The joint table for 2 random variable X & Y is as follows. Determine marginal probability distribution of X & Y also compute
(a) expectations of X, Y & XY
(b) S.D.s of X, Y
(c) covariance of X & Y
(d) correlation of X & Y

Sol: marginal distribution of X

x_i	1	2
$f(x_i)$	0.6	0.4

marginal distribution of Y

y_i	-2	-1	4	5
$g(y_i)$	0.2	0.3	0.1	0.3

(a) $E(X) = \Sigma x_i f(x_i) = 1(0.6) + 2(0.4) = 0.6 + 0.8 = 1.4 = \mu_X$

$E(Y) = \Sigma y_i g(y_i) = -2(0.3) + (-1)(0.3) + 4(0.1) + 5(0.3) = -0.6 - 0.3 + 0.6 + 1.5 = 1 = \mu_Y$

$E(XY) = \Sigma x_i y_i J_{ij} = (1)(-2)(0.1) + (1)(-1)(0.2) + (1)(4)(0) + (1)(5)(0.3) +$
$(2)(-2)(0.2) + (2)(4)(0.1) + (2)(5)(0)$
$= -0.2 + 0.2 + 0 + 1.5 - 0.8 - 0.2 + 0.8 + 0 = 0.9$

$\therefore \mu_X = 1.4,\ \mu_Y = 1,\ E(XY) = 0.9$

(b) $V(X) = \sigma_x^2 = E(x^2) - \mu_x^2$

$E(x)^2 = \Sigma x_i^2 f(x_i) = 1(0.6) + 4(0.4) = 0.6 + 1.6 = 2.2$

$= (2.2) - (1.4)^2 = 0.24$

$\therefore \sigma_x = 0.489 \approx 0.49$

(c) $cov(x,y) = E(xy) - \mu_x \mu_y$
$= (0.9) - (1.4)(1) = -0.5$

$cov(x,y) = -0.5$

(d) $\rho(x,y) = \dfrac{cov(x,y)}{\sigma_x \sigma_y} = \dfrac{-0.5}{(0.49)(3.1)} = -0.329$

$\rho(x,y) = -0.329$

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To verify X & Y are dependent consider $f(x_i)g(y_i)$

ie: $f(x_1)g(y_1) = (0.6)(0.3) = 0.18 \neq J_{11}$ $(\because J_{11} = 0.1)$

III'ly $f(x_2)g(y_3) = (0.4)(0.1) = 0.04 \neq J_{23}$ $(\because J_{23} = 0.1)$

$\therefore$ X & Y are dependent random variables

To find $p(X+Y>0)$ consider $(X+Y>0)$

ie: the pairs $(x_1, y_3) = (1,4)$ $(x_1, y_4) = (1,5)$ $(x_2, y_2) = (2,-1)$

$(x_2, y_3) = (2,4)$ $(x_2, y_4) = (2,5)$

$\therefore p(X+Y>0) = J_{13} + J_{14} + J_{22} + J_{23} + J_{24}$

$= 0 + 0.3 + 0.1 + 0.1 + 0$

$$p(X+Y>0) = 0.5$$

(3) Suppose X & Y are independent random variables with the following respective distribution, find the joint distribution of X and Y also verify that $cov(X,Y) = 0$

x_i	1	2
$f(x_i)$	0.7	0.3

y_i	-2	5	8
$g(y_i)$	0.3	0.5	0.2

Sol^n Since X and Y are independent the joint distribution $J(x,y)$ is given as $f(x_i)g(y_j) = J_{ij}$ and it is obtained by multiplying the marginal distribution of X & Y.

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$\therefore$ J.P.D Table is

X/Y	$y_1 = -2$	$y_2 = 5$	$y_3 = 8$	$f(x_i)$
$x_1 = 1$	0.21	0.35	0.14	0.7
$x_2 = 2$	0.09	0.15	0.06	0.3
$g(y_i)$	0.3	0.5	0.2	0

$Cov(x,y) = E(x,y) - \mu_x \mu_y$ → ①

$\mu_x = E(x) = \sum x_i f(x_i)$

$= 1(0.7) + 2(0.3)$

$\mu_x = 1.3$

$\mu_Y = E(y) = \Sigma y_j g(y_j)$

$= -2(0.3) + 5(0.5) + 8(0.2)$

$\boxed{\mu_Y = 3.5}$

$E(xy) = \Sigma x_i y_j J_{ij} = (1)(-2)(0.21) + (1)(5)(0.35) + (1)(8)(0.14) +$
$(2)(-2)(0.09) + (2)(5)(0.15) + (2)(8)(0.06)$

$\boxed{E(xy) = 4.55}$

$\therefore \text{cov}(x, y) = 4.55 - (3.5)(1.3) = 0$

$\boxed{\therefore \text{cov}(xy) = 0}$

(4) X & Y are independent random variables X take values 2, 5, 7 with probability 1/2, 1/4, 1/4, respectively Y take value 3, 4, 5 with the probability 1/3, 1/3, 1/3

(a) find the JPD of X & Y

(b) Show that the covariance of x and y is equal is zero.

(c) find the Probability distribution $z = x + y$

2015 given the data is

x_i	2	5	7
$f(x_i)$	1/2	1/4	1/4

y_i	3	4	5
$g(y_i)$	1/3	1/3	1/3

(a) Joint probability distribution of x and y

x\y	$y_1 = 3$	$y_2 = 4$	$y_3 = 5$	$f(x_i)$
$x_1 = 2$	1/6	1/6	1/6	1/2
$x_2 = 5$	1/12	1/12	1/12	1/4
$x_3 = 7$	1/12	1/12	1/12	1/4
$g(y_i)$	1/3	1/3	1/3	1

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$\because$ x and y are independent random variables

$\text{Cov}(x, y) = 0$

$cov(x,y) = E(xy) - \mu_x \mu_y \rightarrow (1)$

$\mu_x = E(x) = \Sigma x_i f(x_i) = 2(1/2) + 5(1/4) + 7(1/4) = 4$

$\mu_y = E(y) = \Sigma y_i g(y_i) = 3(1/3) + 4(1/3) + 5(1/3) = 4$

$E(xy) = \Sigma x_i y_i J_{ij} = 2(3)(1/6) + 2(4)(1/6) + 2(5)(1/6) + 5(3)(1/12) + 5(4)(1/12) + 5(5)(1/12) + 7(3)(1/12) + 7(4)(1/12) + 7(5)(1/12)$

$E(xy) = 16$

$cov(x,y) = 16 - 4(4) = 0$

$z = x + y$

let $z_i = x_i + y_i$ $\therefore (z_i) = \{5, 6, 7, 8, 9, 10, 11, 12\}$

$\therefore$ The corresponding probabilities are

$1/6, 1/6, 1/6, 1/12, 1/12 + 1/12 = 1/6, 1/12, 1/12$

$\therefore$ the probability of $z = x + y$ is

z	5	6	7	8	9	10	11	12
P(z)	1/6	1/6	1/6	1/12	1/12	1/6	1/2	1/12

Also $\Sigma p(z) = 1$

(5) the Joint probability distribution of two discrete random variables X & Y is given by $f(x,y) = k(2x+y)$ where x & y are integers such that $0 \le x \le 2$, $0 \le y \le 3$

(a) find the value of the constant k

(b) find the marginal distribution of x & y

(c) show that the random variables x & y are dependent

Soln. given $x = \{x_i\} = \{0, 1, 2\}$ and $y = \{y_j\} = \{0, 1, 2, 3\}$

$f(x,y) = k(2x+y)$ ∴ the joint probability distribution table is

x\y	0	1	2	3	Sum
0	0	k	2k	3k	6k
1	2k	3k	4k	5k	14k
2	4k	5k	6k	7k	22k
Sum	6k	9k	12k	15k	42k

(a) To find k we must have $42k = 1$

$\therefore k = 1/42$

(b) marginal distribution of X

x_i	0	1	2
$f(x_i)$	6/42	14/42	22/42

marginal distribution of Y

y_j	0	1	2	3
$g(y_j)$	6/42	9/42	12/42	15/42

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(b) A fair coin is tossed thrice the random variables X & Y are defined as follows X = 0 or 1, a/c as head or tail occurs on the first toss

Y = No of heads

(a) Determine the distribution of X & Y

(b) Determine the joint distribution of X & Y

(c) Obtain the expectation of X, Y & XY also find SDs of X & Y

(d) Compute covariance & correlation of X & Y

Soln/. the sample space S and the associated random variables X & Y are given as follows:

S	HHH	HHT	HTH	HTT	THH	TTH	THT	TTT
X	0	0	0	0	1	1	1	1
Y	3	2	2	1	2	1	1	0

(a) the probability distributions of X & Y is as follows

$X = \{x_i\} = \{0, 1\}$ $Y = \{y_j\} = \{0, 1, 2, 3\}$

$P(X=0)$ is $\frac{4}{8} = \frac{1}{2}$ $P(X=1)$ is $4/8 = 1/2$

$P(Y=0)$ is $1/8$ $P(Y=1)$ is $3/8$ $P(Y=2) = 3/8$ $P(Y=3) = 1/8$

∴ Marginal distribution of X & Y are

x_i	0	1
$f(x_i)$	1/2	1/2

y_i	0	1	2	3
$g(y_i)$	1/8	3/8	3/8	1/8

(b) the joint probability distribution of X & Y is $J_{ij} = P(X=x_i, Y=y_j)$

$x_1 = 0$ $x_2 = 1$ & $y_1 = 0$, $y_2 = 1$, $y_3 = 2$, $y_4 = 3$

$J_{11} = P(X=0, Y=0) = 0$

$J_{12} = P(X=0, Y=1) = 1/8$ (i.e : HTT)

$J_{13} = P(X=0, Y=2) = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$ (ie: HHT, HTH)

$J_{14} = P(X=0, Y=3) = \frac{1}{8}$ (ie; HHH)

$J_{21} = P(X=1, Y=0) = \frac{1}{8}$ (ie; TTT)

$J_{22} = P(X=1, Y=1) = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$ (ie: TTH, THT)

$J_{23} = P(X=1, Y=2) = \frac{1}{8}$ (ie; THH)

$J_{24} = P(X=1, Y=3) = 0$

x\y	0	1	2	3	Sum
0	0	1/8	1/4	1/8	1/2
1	1/8	1/4	1/8	0	1/2
Sum	1/8	3/8	3/8	1/8	1

(c) $E(X) = \mu_X = \sum x_i f(x_i) = 0(\frac{1}{2}) + 1(\frac{1}{2}) = \frac{1}{2}$

$E(Y) = \mu_Y = \sum y_j g(y_j) = 0(\frac{1}{8}) + 1(\frac{3}{8}) + 2(\frac{3}{8}) + 3(\frac{1}{8}) = \frac{12}{8} = \frac{3}{2}$

$E(XY) = \sum x_i y_j J_{ij} = 0(0)(0) + 1(0)(\frac{1}{8}) + (1)(1)(\frac{1}{4}) + (1)(2)(\frac{1}{8})$
$+ (1)(3)(0)$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$E(X^2) = \sum x_i^2 f(x_i) = 0 + (1)(\frac{1}{2}) = \frac{1}{2}$

$\sigma_x^2 = E(X^2) - \mu_x^2$

$= \frac{1}{2} - (\frac{1}{2})^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$E(Y^2) = \sum y_j^2 g(y_j) = 0 + (1)(\frac{3}{8}) + 4(\frac{3}{8}) + 9(\frac{1}{8})$

$= \frac{3}{8} + \frac{3}{2} + \frac{9}{8} = \frac{3+12+9}{8} = \frac{24}{8}$

$E(Y^2) = 3$

$\sigma_y^2 = E(Y^2) - \mu_y^2$

$= 3 - (\frac{3}{2})^2$

$= 3 - \frac{9}{4}$

$= \frac{12-9}{4} = \frac{3}{4}$

$\therefore$ S.D of x is $\sigma_x = \sqrt{\frac{1}{4}} = \frac{1}{2}$

S.D of Y is $\sigma_y = \sqrt{\frac{3}{4}} = \sqrt{3}/4$

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(d) $Cov(x,y) = E(xy) - \mu_x \mu_y$

$= \frac{1}{2} - (\frac{1}{2})(\frac{3}{2})$

$= \frac{1}{2} - \frac{3}{4}$

$$\rho(x,y) = \frac{Cov(x,y)}{\sigma_x \sigma_y}$$

$$= \frac{-\frac{1}{4}}{(\frac{1}{2})(\frac{\sqrt{3}}{2})}$$

$$= -\frac{1}{4} \times \frac{4}{\sqrt{3}}$$

$$\boxed{\rho(x,y) = -\frac{1}{3}}$$

(1) The joint distribution of two random variables X and Y is as follows:

x\y	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

(a) $E(x)$, $E(y)$, $E(x,y)$ (b) σ_x & σ_y (c) $Cov(x,y)$ (d) $\rho(x,y)$

Soln: Distribution of x:

x_i	1	5
$f(x_i)$	½	½

Distribution of y:

y_i	-4	2	7
$g(y_i)$	3/8	3/8	1/4

(a) $E(x) = \sum x_i f(x_i) = (1)(\frac{1}{2}) + (5)(\frac{1}{2}) = 3$

$E(y) = \sum y_i g(y_i)$

$= (-4)(3/8) + 2(3/8) + (7)(1/4) = 1$

$\boxed{\mu_x = E(x) = 3 \text{ and } \mu_y = E(y) = 1}$

(b) $E(XY) = \sum x_i y_j J$

$= (1)(-4)(1/8) + (1)(2)(1/4) + (1)(7)(1/8)$
$+ (5)(-4)(1/4) + (5)(2)(1/8) + (5)(7)(1/8)$

$= \frac{-1}{2} + \frac{1}{2} + \frac{7}{8} - 5 + \frac{5}{4} + \frac{35}{8} = \frac{3}{2}$

thus $E(X, Y) = 3/2$

(c) $\sigma^2_{(x)} = E(X^2) - \mu^2_x$ and $\sigma^2_y = E(Y^2) - \mu_y^2$

$E(X^2) = \sum x_i^2 f(x_i)$

$E(x^2) = (1)(1/2) + (25)(1/2) = 13$

$E(y^2) = \sum y_i^2 g(y_i)$

$E(y^2) = 16(3/8) + (4)(3/8) + (49)(1/4) = 79/4$

$\sigma^2_x = 13 - (3)^2 = 4$; $\sigma^2_x = (79/4) - (1)^2 = 75/4$

$\sigma_x = 2$ and $\sigma_y \sqrt{75/4} = 4.33$

(d) $\text{Cov}(x, y) = E(XY) - \mu_x \mu_y$

$= (3/2) - (3)(1) = -3/2$

$\text{Cov}(x, y) = -3/2$

(e) $\rho(x, y) = \frac{\text{Cov}(x, y)}{\sigma_x \sigma_y}$

$= \frac{-3/2}{(2)\sqrt{75/4}} = \frac{-3}{2\sqrt{75}}$

$\rho(x, y) = -0.1732$

10/10

10/10

14/1/22

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Assignment / Quiz / Class Test

Certificate

This is to certify that Mr. / Ms. ANSHADH A

bearing USN. *No.* 1GV20ME401 *has satisfactorily completed the course of Tests and assignments as prescribed by Visvesvaraya Technological University for* IV *Semester B.E./ M.Tech, Degree in* Mining *Branch / Specialization for the academic year* 2020-21 *for the Subject* TDFM *and Code* 18MNA6

For Departmental Use Only :

Date	Particulars	Max Marks	Marks obtained	Signature of Faculty	Signature of Student
25.05.21	A/~~Q/~~CT	10	10	[signature]	[signature]
15.07.21	A/Q/~~CT~~	10	10	[signature]	[signature]
06.8.21	A/~~Q/~~CT	10	10	[signature]	[signature]

[signature] 14/1/22
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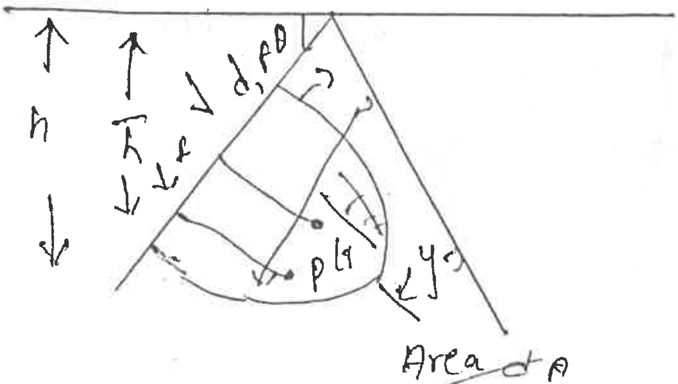
III – ASSIGNMENT

Q) Derive an expression for total pressure & center of pressure for a inclined plane surface submerged in liquid.

→ Consider a plane surface of arbitrary shape immersed in a liquid in such a way that the plane of the surface makes an angle θ with the free surface of the liquid as shown in fig.



Let A = Total area of inclined surface

$\bar{h}$ = Depth of C.G of inclined area from surface

h' = Distance of center of pressure from free surface of liquid.

θ = Angle made by the plane of the surface with free liquid surface

Total pressure force on the whole area,

$F = \int dF = \int \rho g h$ But from fig

$$\frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h}{y_1} \sin\theta$$

$h = y \sin\theta$

$f = \int \rho g \times y \times \sin\theta \times \delta A = \rho g \sin\theta \int y \, dA$

But $\int y \, dA = A\bar{y}$

where $\bar{y}$ = Distance of C.G from axis O-O

$\therefore f = \rho g \sin\theta$

Centre of pressure (h)

Pressure force on the strip $\delta F = \rho g h \delta A$

$= \rho g y \sin\theta \, dA$

moment of the force δF, about axis O-O

$= \delta F \times y = \rho g y \sin\theta \, \delta A \times y = \rho g \sin\theta$

6) Define Buoyancy. Explain about meta center & meta centre height.

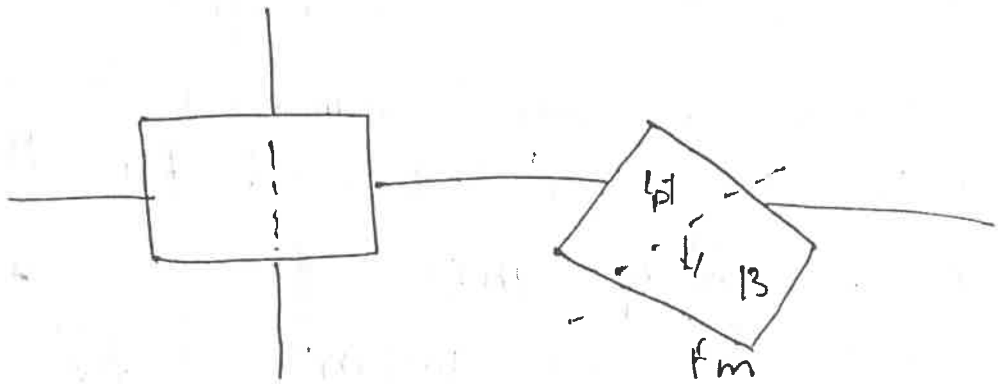
=) when a body is immersed in a fluid an upward force is exterted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body & is called buoyancy.

meta-centre.

*) It is defined as the point about which a body start oscillating when the body is tilted by a small angle.

x The meta centre may also defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

*) consider a body floating in a liquid as shown in fig



2) Let the body is given a small angular displacement in the clockwise direction as shown in fig.

3) Derive Bernoulli's eq from euler's eq of motion & write the assumption made.

→ Bernoulli's eq^n is obtained by integrating the euler's eq^n of motion.

$$\int \frac{dp}{\rho} + \int g\,dz + \int v\,dv = \text{constant}.$$

If flow is incompressible, ρ is constant

$$\therefore \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}.$$

(or)

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant} \rightarrow (1)$$

(or)

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

Assumption

i) The fluid is ideal

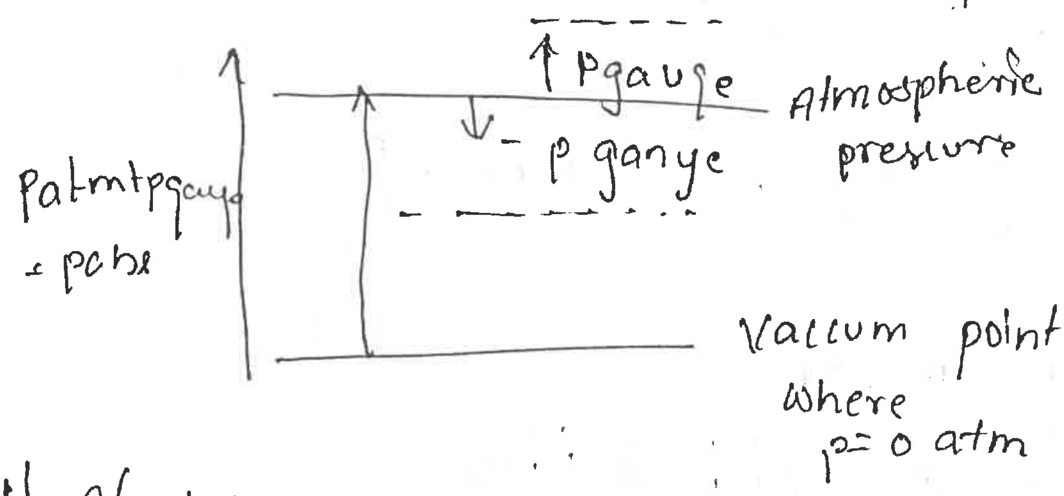
ii) The flow is steady.

iii) The flow is incompressible

iv) The flow is irrotational.

05

8) Differentiate b/w Gauge pressure, Vaccum pressure & absolute pressure.



1) Absolute pressure:-

*) It is the zero reference to a perfect vaccum which exists in the air free space of the universe.

*) It is denoted by Abs. Dervied from latin absolute meaning independent.

2) Gauge pressure.

*) It is the pressure, zero reference against ambient pressure. It is also called over pressure

0.5

3) vaccum pressure.

*) It is the pressure below atmospheric pressure is measured by vaccum gangue

*) It indicates the diffrce b/w atmospheric pressure & absolute pressure

8) Explain the conditions of equilbrium of a floating & submerged bodies.

→ A sub-merged or a floating body is said to be stable if it comes back to its original position after a slight disturbance The relative position of the centre of gravity (G) & centre of boogancy (B) of a body determines the stability of a sub-merged body.

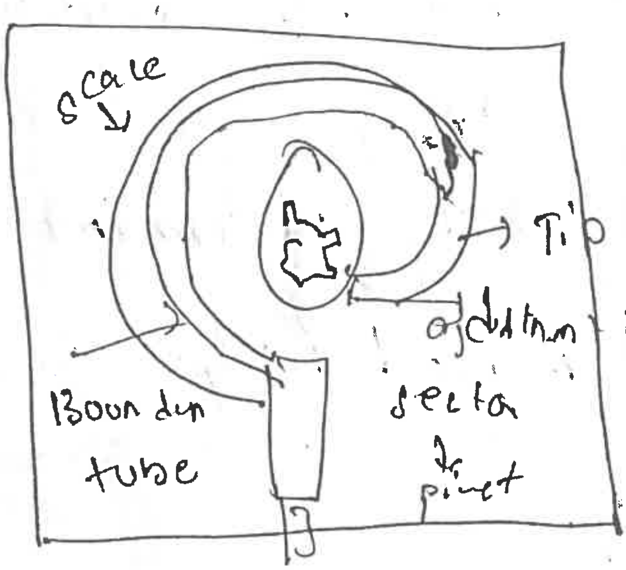
05

1) With a neat sketch explain Bourdon's pressure guage

⇒ working of bourdon pressure guage.

*) Any error due to friction in the spindle bearing is known as lost motion. The mechanical construction has to highly accurate in the case of bourdon tube guage.

*) we consider a cross section of outer edge will have a larger surface than the inner portion, the tube walls will have a thicknes b/w 0.01 & 0.05 inches



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*) The applied pressure acts on the inner walls of the bourdon tube.

*) Due to the applied pressure the bourdon tube tends to change in cross-section from elliptical to circular

7) Explain hydraulic gradient line & total energy line?

⟹ Hydraulic gradient line is basically defined as the line which will give the sum of pressure head ~~& datum~~ head or potential head of a fluid flowing through a pipe with respect to some reference line. Hydraulic gradient line = pressure head + potential head or datum head

$$H.g.l = P/\rho g + z$$

where H.G.L = Hydraulic gradient line

$P/\rho g$ = pressure head.

z = potential head or datum head.

05

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4) Determine meta centric height experimentally.

→ The meta-centric height of a floating can be determined. provide we known the centre of gravity of the floating vessel.

a) floating body.

Let W = weight of vessel including w

G = centre of gravity of the vessel

B = centre of buoyancy of the vessel

The weight w is moved across the vessel towards right through a distance x as shown in fig. The vessel will be title. The angle of distance x a as shown is measured by means a plumline & a protractor attached on the vessel.

1) Explain the concept of pressure measurement by simple manometer?

→ A simple manometer consists of a glass tube having one of its ends connected to a points where pressure is to be measured & other end remain open to atmosphere.

common types of simple manometers are

1) piezometer
2) U-tube manometer &
3) simple column manometer

1) piezometer: It is simplest form to manometer used for measuring gauge pressure

ii) one end of this manometer is connected to the point where pressure is to be measured & other end is open to the atmosphere as shown in fig

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$P \times g \times h \ \frac{N}{m^2}$

2) U- tube-monometer

*) It consists of glass tube bent in U shape one end of which is connected to a point at which pressure is to be measured of the end remain open to the atmosphere.

*) The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.

3) Single column monometer

It a modified form of a u-tube manometer in which one side is a large reservoir & the other side is a small tube & open to the atmosphere

There are two types of single column manometer

1) Vertical single column manometer
2) Inclined single column manometer.

30/30 = 10/10

10/1/22